جمهورية مصر العربية



وزارة التربية والتعليم
والتعليم الفني

نموذج إجابة
امتحان شهادة إتمام الدراسة الثانوية العامة
للعام الدراسى ٢٠١٦/٢٠١٧ - الدور الأول

المـادة : الجبر والهندسة الفراغية (باللغة الانجليزية)

نموذج

أ

المسائل من ← إلى	الدرجة
١ ← ٥	٧
٦ ← ٨	٥
٩ ← ١١	٦
١٢ ← ١٥	٥
١٦ ← ١٩	٧
المجموع	٣٠

لكل المجموع مقدر ومراجع



1-

Answer : ⓒ $^6C_2 + {}^6C_3$ △1

2-

Answer: ⓑ 4 △1

3-

Answer: ⓒ T_6 △1

4-

$\because T_3 = {}^nC_2 \times x^2 = 17 \longrightarrow$ (1) $\frac{1}{2}$

$T_2 \times T_4 = \frac{544}{3}$ divide by T_3

$n \times x \times \frac{T_4}{T_3} = \frac{544}{3 \times 17}$ $\frac{1}{2}$

$n \times x \times \frac{n-3+1}{3} \times x = \frac{32}{3}$

$n\, x^2 (n-2) = 32 \longrightarrow$ (2) $\frac{1}{2}$

from (1) & (2) by divisim

$\frac{n(n-1)\, x^2}{2n\, x^2 (n-2)} = \frac{17}{32}$ $\frac{1}{2}$

$\frac{n-1}{n-2} = \frac{34}{32}$

$\frac{n-1}{n-2} = \frac{17}{16}$

$17n - 34 = 16n - 16$ $\frac{1}{2}$

$n = 18$

Substitude in (2)

$18 \times x^2 \times 16 = 32$

$x^2 = \frac{1}{9} \Rightarrow x = \pm\frac{1}{3}$ $\frac{1}{2}$

Another solution:

$^{n}C_2 (x)^2 = 17$

$3(^{n}C_1 \times x)(^{n}C_3 \times x^3) = 544$

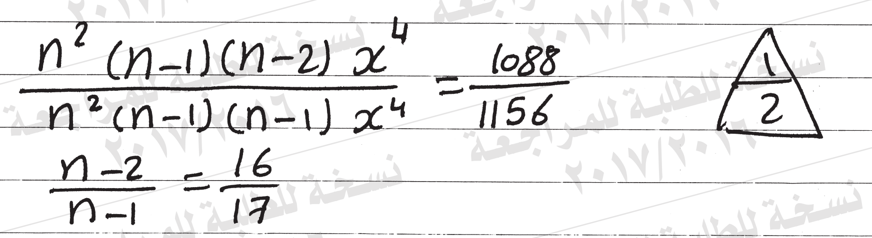
$\frac{n(n-1)}{2} x^2 = 17$

$n(n-1)x^2 = 34 \longrightarrow (1)$ $\frac{1}{2}$

$3nx \times \frac{n(n-1)(n-2)}{6} x^3 = 544$

$n^2(n-1)(n-2)x^4 = 1088 \longrightarrow (2)$ $\frac{1}{2}$

From (1) & (2)



$\frac{n^2(n-1)(n-2)x^4}{n^2(n-1)(n-1)x^4} = \frac{1088}{1156}$ $\frac{1}{2}$

$\frac{n-2}{n-1} = \frac{16}{17}$

$17n - 34 = 16n - 16$

$n = 18$ $\frac{1}{2}$

sub. in (1) $18(18-1)x^2 = 34$

$18 \times 19\ x^2 = 34$ $\frac{1}{2}$

$9x^2 = 1$

$x^2 = \frac{1}{9}$

$x = \pm\frac{1}{3}$ $\frac{1}{2}$

5-

Answer: (b) ω 1

6-

Answer: (c) 1 [1]

7-

Answer: (b) $\frac{-\pi}{4}$ [1]

8-

(a) $r=\sqrt{2}$, $\tan\theta=1$ $\therefore \theta=\frac{\pi}{4}$ [½]

$\therefore Z=\sqrt{2}\left[\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right]$ [½]

$Z^{\frac{1}{3}}=2^{\frac{1}{6}}\left[\cos\frac{\frac{\pi}{4}+2\pi n}{3}+i\sin\frac{\frac{\pi}{4}+2\pi n}{3}\right]$ [½]

such that: $n=0,1,2$

at $n=0$ $Z_1=2^{\frac{1}{6}}\left[\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right]=2^{\frac{1}{6}}e^{\frac{\pi}{12}i}$ [½]

at $n=1$ $Z_2=2^{\frac{1}{6}}\left[\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right]=2^{\frac{1}{6}}e^{\frac{3\pi}{4}i}$ [½]

at $n=2$ $Z_3=2^{\frac{1}{6}}\left[\cos\frac{-7\pi}{12}+i\sin\frac{-7\pi}{12}\right]=2^{\frac{1}{6}}e^{\frac{-7\pi}{12}i}$ [½]

(b) $r=\sqrt{1^2+(-\sqrt{3})^2}=2$ [½]

$\tan\theta=\frac{-\sqrt{3}}{1}=-\sqrt{3}$ [½]

1 θ r $\sqrt{3}$

$\therefore \theta=-60^\circ$ [½]

$\therefore Z=2\left[\cos(-60^\circ)+i\sin(-60^\circ)\right]$ [½]

$Z^{\frac{3}{2}}=2^{\frac{3}{2}}\left[\cos(-60^\circ)+i\sin(-60^\circ)\right]^{\frac{3}{2}}$ [½]

$Z^{\frac{3}{2}}=2\sqrt{2}\left[\cos(-90^\circ)+i\sin(-90^\circ)\right]$ [½]

$Z^{\frac{3}{2}}=2\sqrt{2}\left[\cos(90^\circ)+i\sin(90^\circ)\right]$

9-

$$\xrightarrow{C_2 - C_1 \ \& \ C_3 - C_1} \begin{vmatrix} 1 & 0 & 0 \\ x & y-x & 0 \\ x & 0 & -y-x \end{vmatrix}$$ (1)

$$= 1 \times (y-x)(-y-x)$$ ($\frac{1}{2}$)

$$= -(y-x)(y+x)$$

$$= -(y^2 - x^2)$$

$$= x^2 - y^2$$ ($\frac{1}{2}$)

10-

Answer: (c) $(x-2)^2+y^2+z^2=4$ $\left(1\right)$

11-

let $A=\begin{pmatrix}2 & -3 & -1\\ 1 & 2 & 3\\ 1 & 0 & -2\end{pmatrix}$

$|A|=\begin{vmatrix}2 & -3 & -1\\ 1 & 2 & 3\\ 1 & 0 & -2\end{vmatrix}=2\times-4+3\times-5-1\times-2$ $\left(\frac{1}{2}\right)$

$=-21 \neq 0$

$\therefore$ Rank(A) = 3

The cofactors matrix $=\begin{pmatrix}\begin{vmatrix}2 & 3\\ 0 & -2\end{vmatrix} & -\begin{vmatrix}1 & 3\\ 1 & -2\end{vmatrix} & \begin{vmatrix}1 & 2\\ 1 & 0\end{vmatrix}\\ -\begin{vmatrix}-3 & -1\\ 0 & -2\end{vmatrix} & \begin{vmatrix}2 & -1\\ 1 & -2\end{vmatrix} & -\begin{vmatrix}2 & -3\\ 1 & 0\end{vmatrix}\\ \begin{vmatrix}-3 & -1\\ 2 & 3\end{vmatrix} & -\begin{vmatrix}2 & -1\\ 1 & 3\end{vmatrix} & \begin{vmatrix}2 & -3\\ 1 & 2\end{vmatrix}\end{pmatrix}$ $\left(\frac{1}{2}\right)$

$=\begin{pmatrix}-4 & 5 & -2\\ -6 & -3 & -3\\ -7 & -7 & 7\end{pmatrix}$ $\left(\frac{1}{2}\right)$

$\therefore$ Adj(A) $=\begin{pmatrix}-4 & -6 & -7\\ 5 & -3 & -7\\ -2 & -3 & 7\end{pmatrix}$ $\left(\frac{1}{2}\right)$

$\therefore \begin{pmatrix}x\\ y\\ z\end{pmatrix}=\frac{-1}{21}\times\begin{pmatrix}-4 & -6 & -7\\ 5 & -3 & -7\\ -2 & -3 & 7\end{pmatrix}\times\begin{pmatrix}9\\ 15\\ 12\end{pmatrix}$ $\left(\frac{1}{2}\right)$

$\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\frac{-1}{21}\begin{pmatrix}-210\\ -84\\ 21\end{pmatrix}$

$\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}10\\ 4\\ -1\end{pmatrix}$ $\left(\frac{1}{2}\right)$

$\therefore x=10$, $y=4$, $z=-1$

12-



Answer: © (4, 1, -1)

13-

Answer: © $85^\circ 4'$



14-

Answer: (d) 6 [1]

15-

[1] $\overrightarrow{AB} \cdot \overrightarrow{AC} = AB \times AC \times \cos(\angle BAC)$ [$\frac{1}{2}$]

$= 6 \times 10 \times \frac{6}{10}$

$= 36$ [$\frac{1}{2}$]

[2] The component of $\overrightarrow{CD}$ in the direction of $\overrightarrow{BC} = \frac{\overrightarrow{CD} \cdot \overrightarrow{BC}}{\|\overrightarrow{BC}\|} =$ Zero [$\frac{1}{2}$]

because they are perpendicular. [$\frac{1}{2}$]

[b] $\because \cos^2\theta_1 + \cos^2\theta_2 + \cos^2\theta_3 = 1$ [$\frac{1}{2}$]

$\because$ The angles are equal $= \theta$

$\therefore 3\cos^2\theta = 1$ [$\frac{1}{2}$]

$\therefore \cos^2\theta = \frac{1}{3}$

$\cos\theta = \pm \frac{1}{\sqrt{3}}$

$\therefore \overrightarrow{A} = \|\overrightarrow{A}\| \left[\cos\theta\, \overrightarrow{i} + \cos\theta\, \overrightarrow{j} + \cos\theta\, \overrightarrow{k}\right]$ [$\frac{1}{2}$]

$= 21\sqrt{3}\left[\pm\frac{1}{\sqrt{3}}\overrightarrow{i} \pm \frac{1}{\sqrt{3}}\overrightarrow{j} \pm \frac{1}{\sqrt{3}}\overrightarrow{k}\right]$ [$\frac{1}{2}$]

$\overrightarrow{A} = \pm\left[21\overrightarrow{i} + 21\overrightarrow{j} + 21\overrightarrow{k}\right]$



16-

Answer: (b) $Z = 3$ [1]

17-

Answer: (c) $\left(\frac{-1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$ [1]

18-

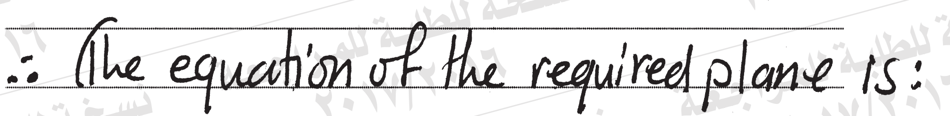
$\because$ The plane contains the st. line L_1 [$\frac{1}{2}$]

$\therefore$ the point $A(0, 3, -5) \in$ the plane.

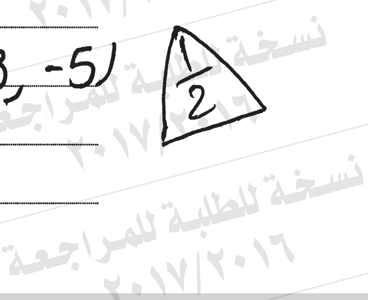
$\because$ the plane // the straigh line L_2 whose unit vector is $(1, -3, 3)$ [$\frac{1}{2}$]



$\therefore$ the vector $(1, -3, 3) \perp$ the required plane [$\frac{1}{2}$]



$\therefore$ The equation of the required plane is:



$(1, -3, 3) \cdot \vec{r} = (1, -3, 3) \cdot (0, 3, -5)$ [$\frac{1}{2}$]

$\therefore x - 3y + 3z + 24 = 0$

19-

The equation is : $\frac{x}{4} + \frac{y}{6} + \frac{z}{3} = 1$

$\therefore$ The points are $A(4,0,0)$, $B(0,6,0)$, $C(0,0,3)$ $\frac{1}{2}$

$\overrightarrow{AB} = \vec{B} - \vec{A} = (0,6,0) - (4,0,0) = (-4,6,0)$ $\frac{1}{2}$

$\overrightarrow{AC} = \vec{C} - \vec{A} = (0,0,3) - (4,0,0) = (-4,0,3)$ $\frac{1}{2}$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -4 & 6 & 0 \\ -4 & 0 & 3 \end{vmatrix}$$ $\frac{1}{2}$

$$= 18\vec{i} + 12\vec{j} + 24\vec{k}$$

$\therefore$ area of triangle $= \frac{1}{2} \| \overrightarrow{AB} \times \overrightarrow{AC} \|$ $\frac{1}{2}$

$$= \frac{1}{2}\sqrt{18^2 + 12^2 + 24^2}$$

$$= \sqrt{261} = 3\sqrt{29}$$ square unit $\frac{1}{2}$

Another solution

The equation: $\frac{x}{4} + \frac{y}{6} + \frac{z}{3} = 1$ $\frac{1}{2}$

$\therefore$ The points are : $A(4,0,0)$, $B(0,6,0)$, $C(0,0,3)$ $\frac{1}{2}$

$AB = \sqrt{(4-0)^2+(0-6)^2+(0-0)^2} = \sqrt{52} = 7.2$ length unit
$AC = \sqrt{(4-0)^2+(0-0)^2+(0-3)^2} = \sqrt{25} = 5$ length unit
$BC = \sqrt{(0-0)^2+(6-0)^2+(0-3)^2} = \sqrt{45} = 6.7$ length unit
$\frac{1}{2}$

Area of triangle $= \sqrt{P(P-a)(P-b)(P-c)}$ $\frac{1}{2}$

$P = \frac{1}{2}(a+b+c) = \frac{1}{2}(7.2+5+6.7) = 9.45$ $\frac{1}{2}$

Area of triangle $= \sqrt{9.45(9.45-7.2)(9.45-5)(9.45-6.7)}$

$= 16.1$ square unit. $\frac{1}{2}$

(انتهت الإجابة وتراعى الحلول الأخرى)